CS2080: Applied Privacy for Data Science
Beyond Noise Addition & Synthetic Data

School of Engineering & Applied Sciences
Harvard University

March 5, 2025

Housekeeping

- My (Salil’s) OH moved from tomorrow (surf’s up) to today 3-3:45pm, SEC 3.327.
- Fill out midterm feedback survey by tonight!
- Participation feedback to be released today or tomorrow.
- Contextual Integrity problem moved to hw6.

BEYOND NOISE ADDITION

Beyond Noise Addition

Fact: Laplace (or geometric) mechanism has the optimal worst-case, additive accuracy for answering a single real-valued query with DP.

Q: Why might we want other mechanisms?

Approaches we'll see:

1. Decompose a computation into several queries that can be answered with Laplace (cf. HW4 #2)
2. The exponential mechanism: applies even to discrete outcomes (ex: median)
3. Approaches based on local sensitivity and restricted sensitivity ~~(ex: graph statistics → beyond tabular data)~~

DP Medians

Recall: median over $\mathcal{X} = [0,1]$ has global sensitivity $\geq 1/2$

- Laplace Mechanism is useless.
- Same applies for discrete data, e.g. education variable ∈ $\{\mathrm{bb}, 1,2, \ldots, 16\}$ in PUMS dataset.

Q: what mechanism from prior weeks could use to estimate the median of a discrete variable?

A: DP histograms!

- Add noise $\mathrm{Lap}(2/\varepsilon)$ to each bin
- Compute median of noisy histogram
- ε-DP by post-processing.

Exponential Mechanism

Given candidate space $\mathcal{C}$ and score function $s : \mathcal{X} \times \mathcal{C} \to \mathbb{R}$, the exponential mechanism is:

$$M(x)\text{: output } c \in \mathcal{C} \text{ with probability} \propto \exp\left(\varepsilon \cdot \frac{s(x,c)}{2 \cdot \Delta s}\right),$$

$$\text{where } \Delta s \overset{\text{def}}{=} \max_{x \sim x', c} |s(x,c) - s(x',c)|.$$

$$= \max_{x \sim x'} \|s(x,\cdot) - s(x',\cdot)\|_{\infty}$$

Thm: above mechanism is ε-DP.

Q: what is a good score function for the median?

A: $s(x,c) = -|\#\{i : x_i < c\} - \#\{i : x_i > c\}|$

Why? $\max_c s(x,c) = s\big(x, \mathrm{median}(x)\big) = 0;\ \Delta s = 1$ wrt d_{Sym}

Utility of the Exp Mech Median

Next time: notebook for experiments comparing using exponential mechanism vs. histograms for the median.

Theoretical explanation: whp

- Exp mech outputs c with $s(x,c) \geq -O(\log|\mathcal{C}|)/\varepsilon$.
- Histogram outputs y with $s(x,c) \geq -O(|\mathcal{C}|^{1/2})/\varepsilon$.

Q: How to implement for continuous $\mathcal{C} = [0,1]$?

Local Sensitivity

- Even when $\Delta q = \mathrm{GS}_q$ is large, the local sensitivity may often be small on many natural datasets x:
$$\mathrm{LS}_q(x) \overset{\text{def}}{=} \max_{x' : x' \sim x} |q(x') - q(x)|.$$

- Adding noise proportional to local sensitivity is not DP (why?)

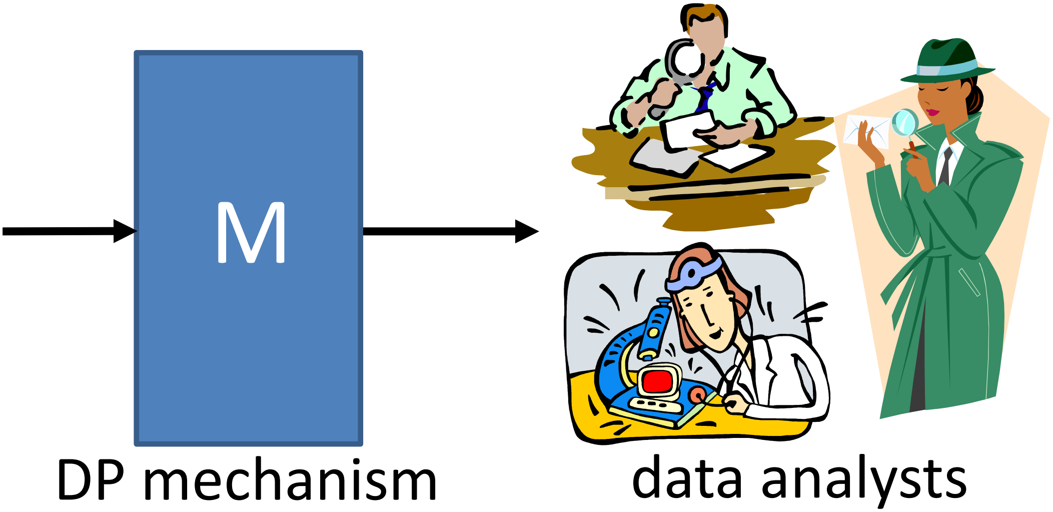
- But there are several DP methods that approximate this idea (smooth sensitivity, propose-test-release, privately bounding local sensitivity, restricted sensitivity) when local sensitivity is small on all “nearby” or “similar” datasets.

- Examples: DP graph analysis (Spr 2022 2/22 slides), HW5 1c

ONE-SHOT RELEASES: SYNTHETIC DATA

One-Shot Releases

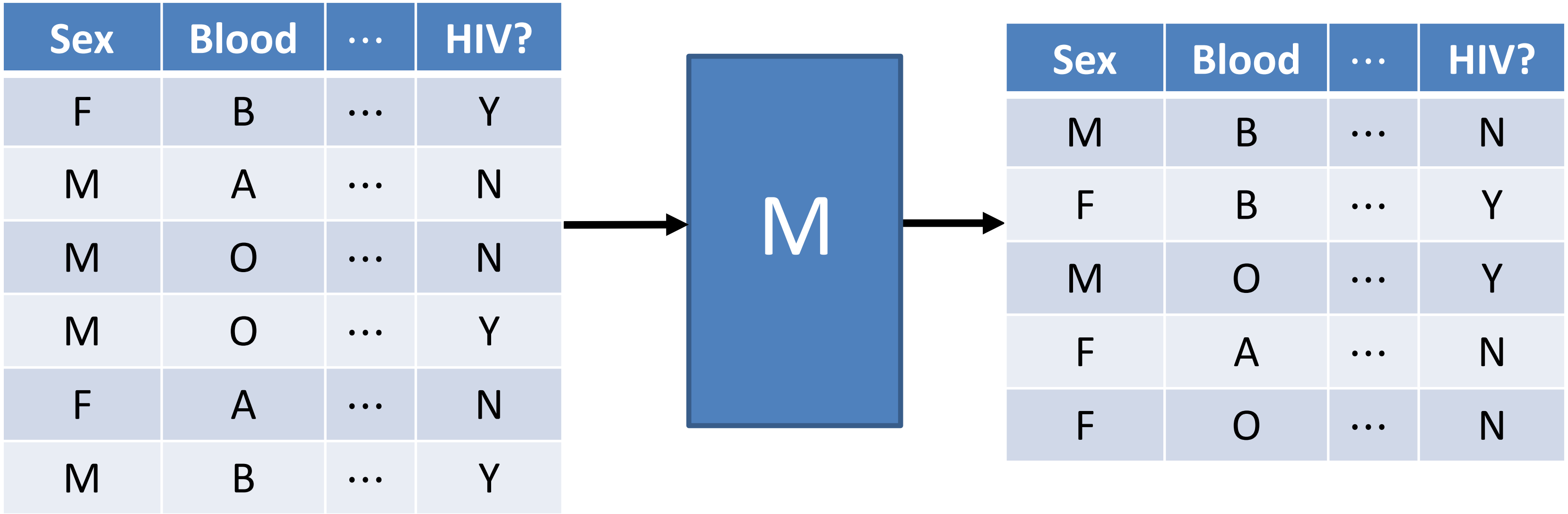
Sex	Blood	…	HIV?
F	B	…	Y
M	A	…	N
M	O	…	N
M	O	…	Y
F	A	…	N
M	B	…	Y



Goal: release as much useful info as possible given privacy budget

- Ideally support unforeseen analyses
- Summary statistics
- ML model
- Synthetic data

Differentially Private Synthetic Data



Sex	Blood	⋯	HIV?
F	B	⋯	Y
M	A	⋯	N
M	O	⋯	N
M	O	⋯	Y
F	A	⋯	N
M	B	⋯	Y

Sex	Blood	⋯	HIV?
M	B	⋯	N
F	B	⋯	Y
M	O	⋯	Y
F	A	⋯	N
F	O	⋯	N

$M\colon \mathcal{X}^n \rightarrow \mathcal{X}^m$ such that:

- $M(x)$ has the same schema as a real dataset.
- $M(x)$ reflects many statistical properties of x.
- M is differentially private.

Synthetic Data via DP Histograms

1. Use singleton bins $B_y = \{y\}$ for each $y \in \mathcal{X}$.
2. Construct a DP histogram $\left(a_1, \dots, a_{|\mathcal{X}|}\right) \leftarrow M_{\text{hist}}(x)$, where $a_y \approx \#\{i : x_i = y\}$.
3. Output synthetic dataset $\hat{x}$ with a_y copies of each element y.

Difficulties?

- a_y's may not be nonnegative integers.
  - Soln 1: use Geometric Mechanism and clamp at 0.
  - Soln 2: use Exponential Mechanism with range $\{0, \dots, n\}$.
- Poor utility & efficiency when $\mathcal{X}$ is large.

Stability-Based Histogram

1. Let $B_1, \dots, B_k \subseteq \mathcal{X}$ be disjoint bins.
2. Define $q_j : \mathcal{X}^n \to \{0,1\}$ by $q_j(x) = \#\{i : x_i \in B_j\}$.
3. For each j s.t. $q_j(x) > 0$:
   a) Let $a_j = q_j(x) + Z_j$ for $Z_j \sim \text{Geo}(2/\varepsilon)$.
   b) If $a_j > \left\lceil \frac{2}{\varepsilon} \cdot \ln \frac{1}{\delta} \right\rceil$, output (j, a_j).
4. Treat all other bins as having a zero count.

Intuition for (ε, δ)-DP:

- Only difference from pure DP is treatment of zero bins.
- If $q_j(x) = 0$, then $q_j(x') \leq 1$ for every $x' \sim x$, and

$$\Pr\left[1 + Z_j > \left\lceil \frac{2}{\varepsilon} \cdot \ln \frac{1}{\delta} \right\rceil\right] < \delta.$$

Stability-Based Histogram

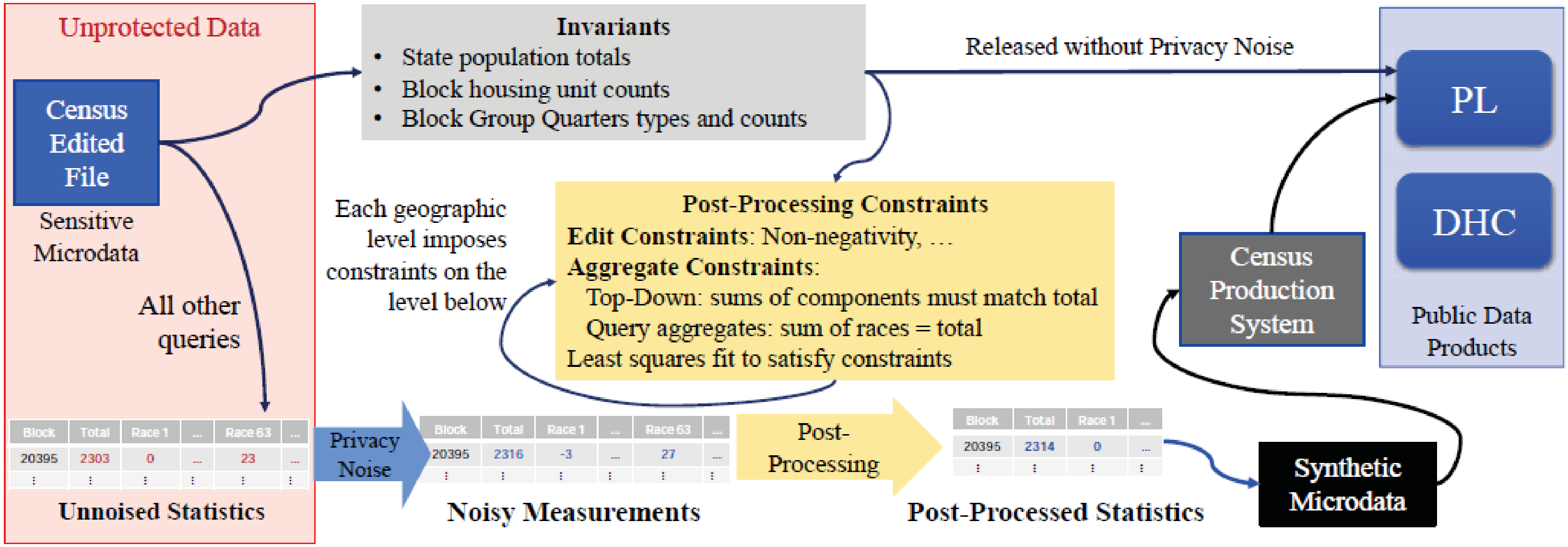
1. Let $B_1, \ldots, B_k \subseteq \mathcal{X}$ be disjoint bins.
2. Define $q_j : \mathcal{X}^n \to \{0,1\}$ by $q_j(x) = \#\{i : x_i \in B_j\}$.
3. For each j s.t. $q_j(x) > 0$:
   a) Let $a_j = q_j(x) + Z_j$ for $Z_j \sim \mathrm{Geo}(2/\varepsilon)$.
   b) If $a_j > \left\lceil \frac{2}{\varepsilon} \cdot \ln \frac{1}{\delta} \right\rceil$, output (j, a_j).
4. Treat all other bins as having a zero count.

Benefits:

- Computation and output size linear in n rather than $|\mathcal{X}|$.
- Max error $O((1/\varepsilon) \cdot \ln(1/\delta))$ whp, independent of $|\mathcal{X}|$.
- But still can have poor utility when $|\mathcal{X}|$ large. (Why?)

CASE STUDY: 2020 U.S. CENSUS

Census DAS Process



Unprotected Data

Census Edited File

Sensitive Microdata

All other queries

Block	Total	Race 1	...	Race 63	...
20395	2303	0	...	23	...
⋮	⋮	⋮	⋮	⋮	⋮

Unnoised Statistics

Invariants

- State population totals
- Block housing unit counts
- Block Group Quarters types and counts

Released without Privacy Noise

Each geographic level imposes constraints on the level below

Post-Processing Constraints

Edit Constraints: Non-negativity, …

Aggregate Constraints:

Top-Down: sums of components must match total

Query aggregates: sum of races = total

Least squares fit to satisfy constraints

Privacy Noise

Block	Total	Race 1	...	Race 63	...
20395	2316	-3	...	27	...
⋮	⋮	⋮	⋮	⋮	⋮

Noisy Measurements

Post-Processing

Block	Total	Race 1	...
20395	2314	0	...
⋮	⋮	⋮	⋮

Post-Processed Statistics

Synthetic Microdata

Census Production System

PL

DHC

Public Data Products

Figure 5-1: Process used to produce privacy-protected data products.

Consistency and Privacy for the 2020 Census 51 January 11, 2022

Consistency & Optimization

- Structural Zeroes: Enforced by edit and imputation, DP can’t reintroduce it
  - Householder and spouse/partner must be at least 15 yrs old
  - Every household must have exactly one householder
  - At least one of the binary race flags must be 1
  - Etc.

- Invariants: public statistics with exact values
  - State population totals
  - Linear constraints: sum of county populations equals state population
  - Single-gender group quarters (dorms, prisons)

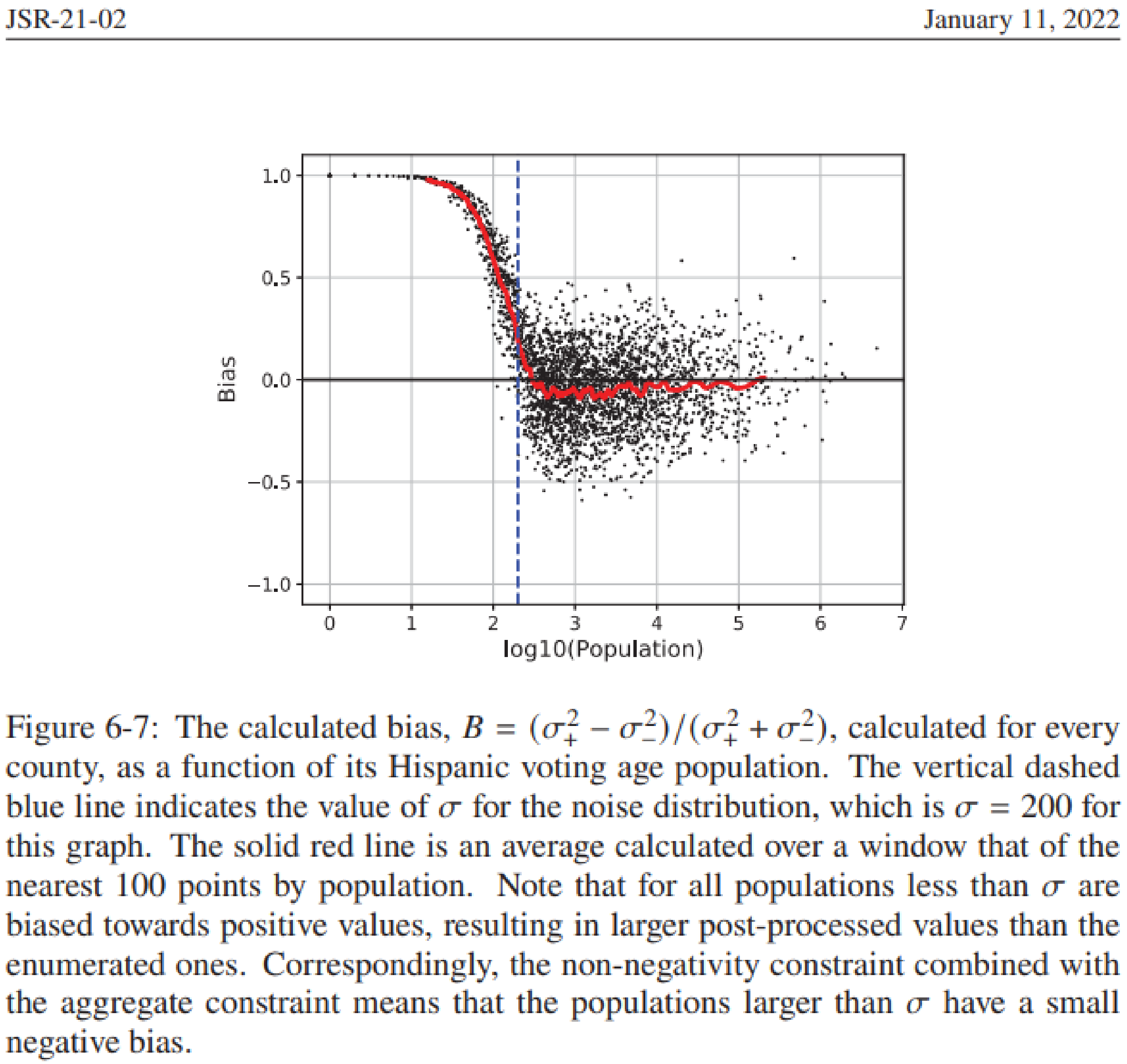
- Optimizing accuracy: for a set $\mathcal{Q}$ of queries
  - Obtain DP answers to a set $\mathcal{Q}$’ of “measurement” queries, then use optimization tools to reconstruct synthetic data to optimize answers on $\mathcal{Q}$.

Census Bureau’s Use of DP

Excerpts from:

Michael Hawes and Michael Ratcliffe. “Understanding the 2020 Census Disclosure Avoidance System: Differential Privacy 201 and the TopDown Algorithm,” Census Webinar, May 13, 2021.

Bias

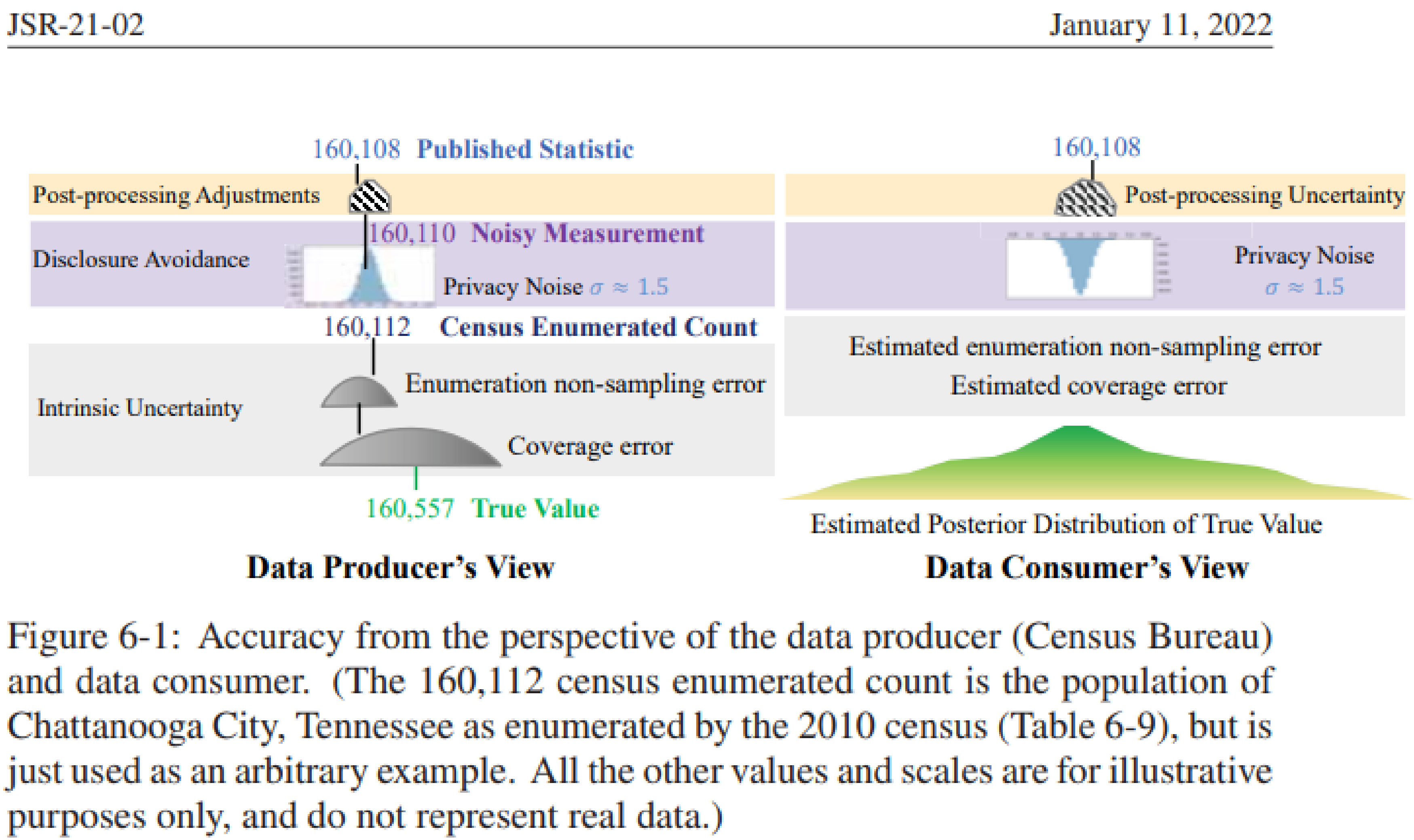


JSR-21-02 January 11, 2022

1.0
0.5
0.0
−0.5
−1.0
Bias
0
1
2
3
4
5
6
7
log10(Population)

Figure 6-7: The calculated bias, $B = (\sigma_+^2 - \sigma_-^2)/(\sigma_+^2 + \sigma_-^2)$, calculated for every county, as a function of its Hispanic voting age population. The vertical dashed blue line indicates the value of σ for the noise distribution, which is $\sigma = 200$ for this graph. The solid red line is an average calculated over a window that of the nearest 100 points by population. Note that for all populations less than σ are biased towards positive values, resulting in larger post-processed values than the enumerated ones. Correspondingly, the non-negativity constraint combined with the aggregate constraint means that the populations larger than σ have a small negative bias.

Data Producer vs. Consumer



JSR-21-02 January 11, 2022

160,108 Published Statistic
Post-processing Adjustments
160,110 Noisy Measurement
Disclosure Avoidance
Privacy Noise $\sigma \approx 1.5$
160,112 Census Enumerated Count
Intrinsic Uncertainty
Enumeration non-sampling error
Coverage error
160,557 True Value
Data Producer's View
160,108
Post-processing Uncertainty
Privacy Noise
$\sigma \approx 1.5$
Estimated enumeration non-sampling error
Estimated coverage error
Estimated Posterior Distribution of True Value
Data Consumer's View

Figure 6-1: Accuracy from the perspective of the data producer (Census Bureau) and data consumer. (The 160,112 census enumerated count is the population of Chattanooga City, Tennessee as enumerated by the 2010 census (Table 6-9), but is just used as an arbitrary example. All the other values and scales are for illustrative purposes only, and do not represent real data.)

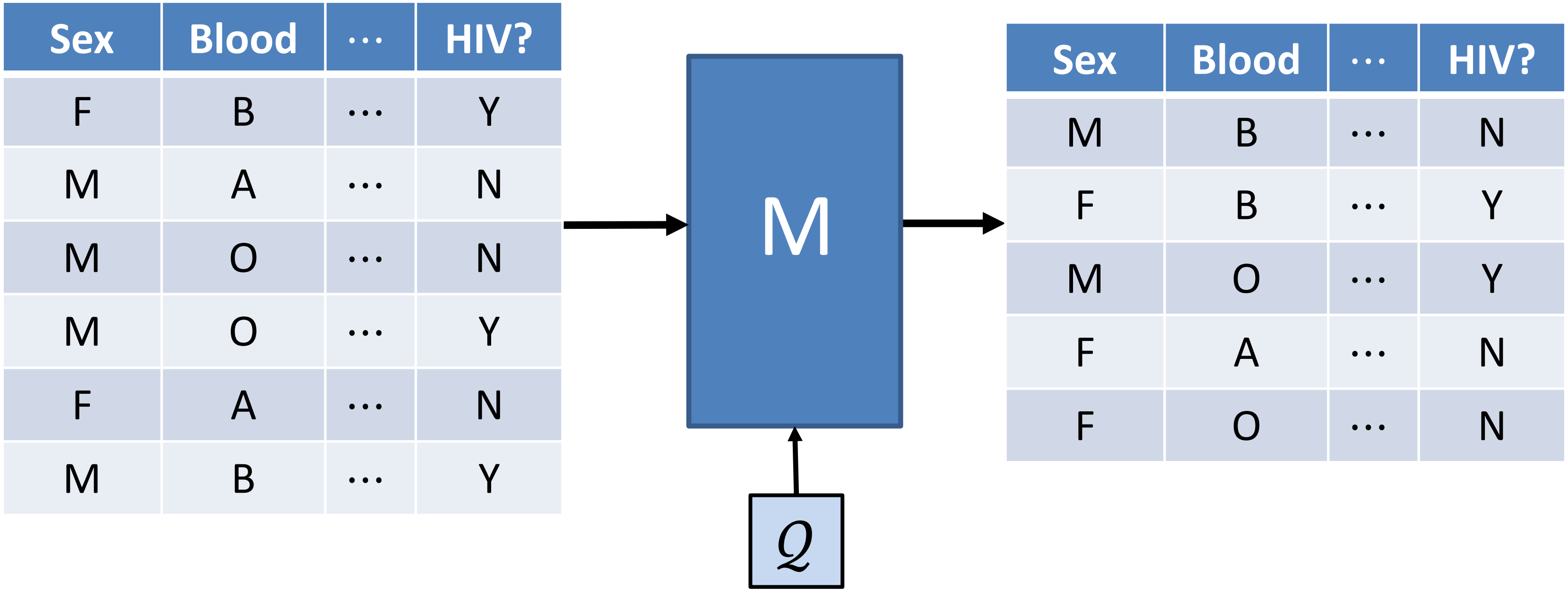
Some challenges raised by JASON

- Detailed queries not used directly in published statistics but consuming privacy-loss budget
- Release of noisy measurements
- Consistency between and within data products
- Threats, risks, and protections not sufficiently quantified
- Better communication with data users
- Re-interpretation of Title 13

STATE OF ART SYNTHETIC DATA GENERATION

Private Multiplicative Weights

[Blum-Ligett-Roth `08,…,Hardt-Rothblum `10]



Sex	Blood	⋯	HIV?
F	B	⋯	Y
M	A	⋯	N
M	O	⋯	N
M	O	⋯	Y
F	A	⋯	N
M	B	⋯	Y

M

$\mathcal{Q}$

Sex	Blood	⋯	HIV?
M	B	⋯	N
F	B	⋯	Y
M	O	⋯	Y
F	A	⋯	N
F	O	⋯	N

(ε,δ)-DP $M\colon \mathcal{X}^n \to \mathcal{X}^m$ such that $\forall q \in \mathcal{Q},\ q\colon \mathcal{X} \to [0,1]$

$$\left|\frac{1}{n}\sum_{i=1}^{n} q(x_i) - \frac{1}{m}\sum_{i=1}^{m} q(M(x)_i)\right| \le O\left(\frac{\sqrt{\log|\mathcal{X}|\cdot\log(1/\delta)}\cdot\log|\mathcal{Q}|}{\varepsilon n}\right)^{1/2}$$

Private Multiplicative Weights

[Hardt-Rothblum `10]

(ε,δ)-DP $M\colon \mathcal{X}^n \to \mathcal{X}^m$ such that $\forall q \in \mathcal{Q},\ q\colon \mathcal{X} \to [0,1]$

$$\left|\frac{1}{n}\sum_{i=1}^{n} q(x_i) - \frac{1}{m}\sum_{i=1}^{m} q(M(x)_i)\right| \leq O\left(\frac{\sqrt{\log|\mathcal{X}|\cdot\log(1/\delta)}\cdot\log|\mathcal{Q}|}{\varepsilon n}\right)^{1/2}$$

Approach:

- DP online learning of a synthetic data distribution, playing against a "query" player trying to distinguish it from dataset

Problem: computation time $\mathrm{poly}(n, |\mathcal{X}|, |\mathcal{Q}|)$.

- Exponential in dimensionality of data and query family.
- Inherent in the worst case (cf. "Complexity of DP").

Practical Approaches

- Use DP queries to learn a model of the data distribution, and use the model to generate synthetic data

- Models (from more structured to less):
  - Multivariate Gaussian (↔ means and (co)variances)
  - Graphical models/Markov Random Fields/Bayes Nets
  - Generative Adversarial Networks

Some Recent Developments

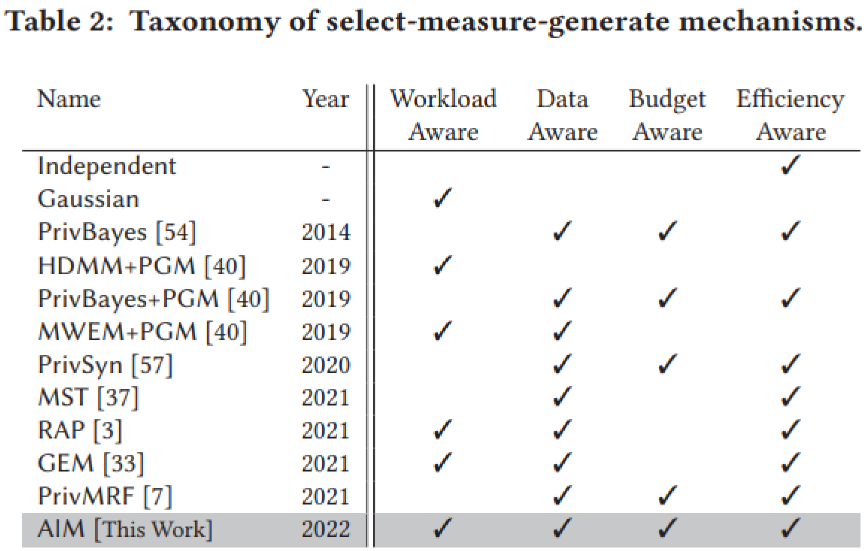


Table 2: Taxonomy of select-measure-generate mechanisms.

Name	Year	Workload Aware	Data Aware	Budget Aware	Efficiency Aware
Independent	-				✓
Gaussian	-	✓			
PrivBayes [54]	2014		✓	✓	✓
HDMM+PGM [40]	2019	✓			
PrivBayes+PGM [40]	2019		✓	✓	✓
MWEM+PGM [40]	2019	✓	✓		
PrivSyn [57]	2020		✓	✓	✓
MST [37]	2021		✓		✓
RAP [3]	2021	✓	✓		✓
GEM [33]	2021	✓	✓		✓
PrivMRF [7]	2021		✓	✓	✓
AIM [This Work]	2022	✓	✓	✓	✓

McKenna et al. “AIM: An Adaptive & Iterative Mechanism…” 2022

- Vietri et al. “Private Synthetic Data for Multitask Learning…” 2022.
- Liu et al. “Generating Private Synthetic Data with Genetic Algorithms” 2023.