CS208: Applied Privacy for Data Science
Reidentification & Reconstruction Attacks

School of Engineering & Applied Sciences
Harvard University

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Announcements

- Fill out [first-class survey](https://shorturl.at/jSosl) if you haven't already: https://shorturl.at/jSosl

- Post questions to Ed rather than emailing us individually.
  Keep an eye on Ed for announcements!

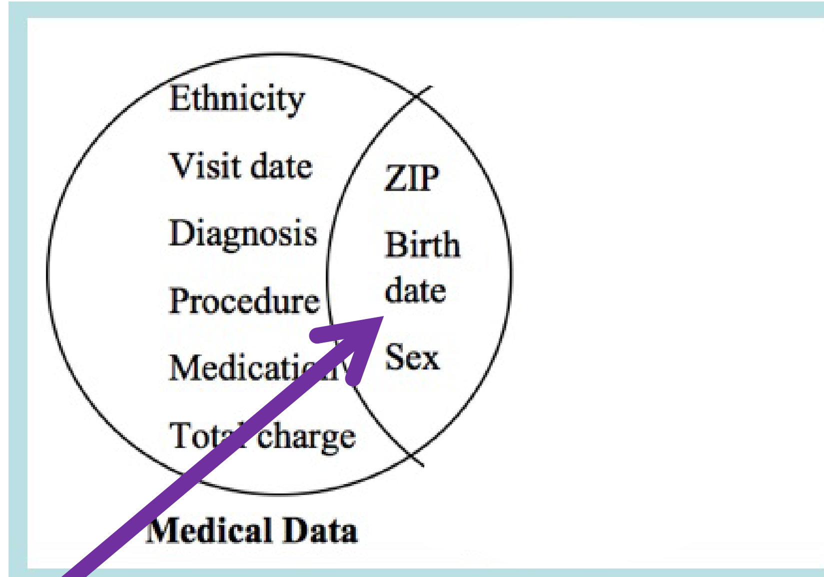
- Let us know ASAP if you can't access course platforms (esp. Ed, Perusall).

- Office hours the rest of this week:
  - Salil Fri 10:30am-12pm (SEC 3.327)
  - Priyanka Wed 2:30pm-4:30pm (SEC 2.101)
  - Zach Thu 3pm-4pm (SEC 3.314)

- Probability/algorithms/stats review sessions this week:
  - Jason Wed 3pm-4pm, Science Center 304
  - Zach Thu 9:45-11:00am, SEC 4.308+Zoom+recording (possibly including programming)

Reidentification via Linkage

Name	Sex	Blood	⋯	HIV?
Chen	F	B	⋯	Y
Jones	M	A	⋯	N
Smith	M	O	⋯	N
Ross	M	O	⋯	Y
Lu	F	A	⋯	N
Shah	M	B	⋯	Y



[Sweeney `97]

Uniquely identify > 60% of the US population [Sweeney `00, Golle `06]

Deidentification via Generalization

- Def (generalization): A generalization mechanism is an algorithm A that takes a dataset $x = (x_1, \ldots, x_n) \in \mathcal{X}^n$ and outputs $A(x) = (T_1, \ldots, T_n)$ where $x_i \in T_i \subseteq \mathcal{X}$ for all i.

- Example:

Name	Sex	Blood	⋯	HIV?
*	F	B	⋯	Y
*	M	A	⋯	N
*	M	O	⋯	N
*	M	O	⋯	Y
*	F	A	⋯	N
*	M	B	⋯	Y

$$T_i = \{\text{all strings}\} \times \{x_{i2}\} \times \cdots \times \{x_{im}\}$$

K-Anonymity [Sweeney `02]

- Def (generalization): A generalization mechanism A satisfies k-anonymity (across all fields) if for every dataset $x = (x_1, \dots, x_n) \in \mathcal{X}^n$ the output $A(x) = (T_1, \dots, T_n)$ has the property that every set T that occurs at all occurs at least k times.

- Example: 3-anonymizing a dataset

$x =$

ZIP	Income	COVID
91010	\$125k	Yes
91011	\$105k	No
91012	\$80k	No
20037	\$50k	No
20037	\$20k	No
20037	\$25k	Yes

$\xrightarrow{A}$

ZIP	Income	COVID
9101⋆	\$75–150k	⋆
9101⋆	\$75–150k	⋆
9101⋆	\$75–150k	⋆
20037	\$0–75k	⋆
20037	\$0–75k	⋆
20037	\$0–75k	⋆

$= A(x)$

Quasi-Identifiers

- Typically, k-anonymity only applied on "quasi-identifiers" – attributes that might be linked with an external dataset. i.e. $\mathcal{X} = \mathcal{Y} \times \mathcal{Z}$, where $\mathcal{Y}$ is domain of quasi-identifiers, and $T_i = U_i \times V_i$, where each U_i occurs at least k times.

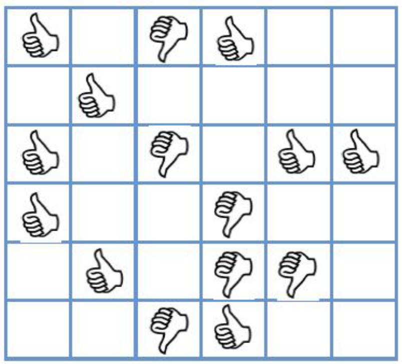
Zip code	Age	Nationality	Condition
130**	<30	*	AIDS
130**	<30	*	Heart Disease
130**	<30	*	Viral Infection
130**	<30	*	Viral Infection
130**	≥40	*	Cancer
130**	≥40	*	Heart Disease
130**	≥40	*	Viral Infection
130**	≥40	*	Viral Infection
130**	3*	*	Cancer
130**	3*	*	Cancer
130**	3*	*	Cancer
130**	3*	*	Cancer

Q: what could go wrong?

Q: What if no quasi-identifiers?

Netflix Challenge Re-identification

[Narayanan & Shmatikov `08]



Anonymized
NetFlix data

Q: Why would Netflix release such a dataset?

Image credit: Arvind Narayanan

Narayanan-Shmatikov Set-Up

- Dataset: x = set of records r (e.g. Netflix ratings)

- Adversary's inputs:
  - $\hat{x}$ = subset of records from x, possibly distorted slightly
  - aux = auxiliary information about a record $r \in D$ (e.g. a particular user's IMDB ratings)

- Adversary's goal: output either
  - r' = record that is "close" to r, or
  - $\perp$ = failed to find a match

Narayanan-Shmatikov Algorithm

1. Calculate $\mathrm{score}(aux, r')$ for each $r' \in \hat{x}$, as well as the standard deviation σ of the calculated scores.
2. Let $r_1{}'$ and $r_2{}'$ be the records with the largest and second-largest scores.
3. If $\mathrm{score}(aux, r_1{}') - \mathrm{score}(aux, r_2{}') > \phi \cdot \sigma$, output $r_1{}'$, else output $\perp$.

An instantiation:

$$\mathrm{score}(aux, r') = \overbrace{\sum_{a \in \mathrm{supp}(aux)}}^{\text{IMDB movies rated by user}} \overbrace{\mathrm{sim}(aux_a, r'_a)}^{\text{Similarity of rating \& date}} \cdot \overbrace{\frac{1}{\log |\{ r' \in \hat{x} : a \in \mathrm{supp}(r')\}|}}^{\text{Downweight movies watched by many Netflix users}}$$

eccentricity $\phi = 1.5$

Narayanan-Shmatikov Results

- For the $1m Netflix Challenge, a dataset of ~.5 million subscribers' ratings (less than 1/10 of all subscribers) was released (total of ~$100m ratings over 6 years).
- Out of 50 sampled IMBD users, two standouts were found, with eccentricities of 28 and 15.
- Reveals all movies watched from only those publicly rated on IMDB.
- Class action lawsuit, cancelling of Netflix Challenge II.

Message: any attribute can be a "quasi-identifier"

k-anonymity across all attributes?

- Utility concerns?
  - Significant bias even when applied on quasi-identifiers, cf. [Daries et al. `14]
- Privacy concerns?
  - Consider mechanism $A(x)$: if Salil is in x and has tuberculosis, generalize starting with rightmost attribute. Else generalize starting on left.
  - Message: privacy is not only a property of the output, but of the input-output relationships.

Downcoding Attacks [Cohen `21]

$\mathbf{X} =$

ZIP	Income	COVID
91010	$125k	Yes
91011	$105k	No
91012	$80k	No
20037	$50k	No
20037	$20k	No
20037	$25k	Yes

$\mathbf{Y} =$

ZIP	Income	COVID
9101⋆	$75–150k	⋆
9101⋆	$75–150k	⋆
9101⋆	$75–150k	⋆
20037	$0–75k	⋆
20037	$0–75k	⋆
20037	$0–75k	⋆

$\mathbf{Z} =$

ZIP	Income	COVID
91010	$125–150k	⋆
9101⋆	$100–125k	⋆
9101⋆	$75–150k	⋆
20037	$0–75k	No
20037	$0–75k	⋆
20037	$25k	Yes

- Downcoding undoes generalization
- X is the original dataset → Y is a 3-anonymized version
- Z is a **downcoding**: It *generalizes* X and *refines* Y

Slide credit: Aloni Cohen

Cohen's Result

Theorem (informal): There are settings in which **every** minimal, hierarchical k-anonymizer (even enforced on all attributes) enables strong downcoding attacks.

Setting

- A (relatively natural) data **distribution** and **hierarchy**, which depend on k

Strength

- **How many** records are refined? $\Omega(N)$ ($> 3\%$ for $k \leq 15$)
- **How much** are records refined? $3D/8$ (38% of attributes)
- **How often**? w.p. $1 - o(1)$ over a random dataset

Composition Attacks

- [Ganti-Kasiviswanathan-Smith `08]:
  Two k-anonymous generalizations of the same dataset can be combined to be not k-anonymous.

- [Cohen `21]:
  Reidentification on Harvard-MIT EdX Dataset [Daries et al. `14]
  - 5-anonymity enforced separately (a) on course combination, and (b) on demographics + 1 course

EdX Quasi-identifiers

User 17	Year of Birth	Gender	Country	Course 1	Course 2	Course 3	
	2000	F	India	Yes	No	Yes	Enrolled
				5		8	# Posts
				Yes		No	Certificate

{Year of Birth, Gender, Country, Course(i).Enrolled, Course(i).Posts}
for i = 1, . . ., 16

User 17	Year of Birth	Gender	Country	Course 1	Course 2	Course 3	
	2000	F	India	Yes	No	Yes	Enrolled
				5		8	# Posts
				Yes		No	Certificate

{Course(1).Enrolled, Course(2).Enrolled, . . ., Course(16).Enrolled

Slide credit: Aloni Cohen

Failure of Composition

User 17	YoB	Gender	Country	Course 1	Course 2	Course 3	
	2000	F	India	Yes	No	Yes	**Enrolled**
				5		8	**# Posts**
				Yes		No	**Certificate**

If you combine the QIs:

- 7.1% uniques (34,000)
- 15.3% not 5-anonymous

Reidentification carried out using LinkedIn profiles
→ dataset heavily redacted

Slide credit: Aloni Cohen

Reading & Discussion for Next Time

- Q: How should we respond to the failure of de-identification?
- Not assigned: writings claiming that de-identification works (see cs208 annotated bibliography)
- Next: what if we only release aggregate statistics?

Attacks on Aggregate Statistics

- Stylized set-up:
  - Dataset $x \in \{0,1\}^n$.
  - (Known) person i has sensitive bit x_i.
  - Adversary gets $q_S(x) = \sum_{i \in S} x_i$ for various $S \subseteq [n]$.

ID	US?
1	1
2	0
3	0
4	1
$\vdots$	$\vdots$
n	1

- How to attack if adversary can query chosen sets S?

- What if we restrict to sets of size at least $n/10$?

This attack has been used on Israeli Census Bureau!
(see [Ziv `13])

Attacks on Exact Releases

- What if adversary cannot choose subsets, but $q_S(x)$ is released for "innocuous" sets S?
- Example: uniformly random $S_1, S_2, \ldots, S_m \subseteq [n]$ are chosen, and adversary receives:
  $\left(S_1, a_1 = q_{S_1}(x)\right), \left(S_2, a_2 = q_{S_2}(x)\right), \ldots, (S_m, a_m = q_{S_m}(x))$
- Claim: for $m = n$, with prob. $1 - o(1)$ adversary can reconstruct entire dataset!
- Proof?

Example for $n = 5$

$S_1 = \{1,2,3\}$, $a_1 = 2$, $S_2 = \{1,3,4\}$, $a_2 = 1$, $S_3 = \{4,5\}, a_3 = 1$,
$S_4 = \{2,3,4,5\}, a_4 = 3, S_5 = \{1,2,4,5\}, \mathrm{a}_5 = 2$

Unknowns: $x_1, x_2, \dots, x_5$

Equations:

1. $x_1 + x_2 + x_3 = 2$
2. $x_1 + x_3 + x_4 = 1$
3. $x_4 + x_5 = 1$
4. $x_2 + x_3 + x_4 + x_5 = 3$
5. $x_1 + x_2 + x_4 + x_5 = 2$

Unique Solution:

$x_1 = 0$
$x_2 = 1$
$x_3 = 1$
$x_4 = 0$
$x_5 = 1$

Attacks on Approximate Statistics

- What if we release statistics $a_i \approx q_{S_i}(x)$?
- Thm [Dinur-Nissim `03]: given $m = n$ uniformly random sets S_j and answers a_j s.t. $\left|a_j - q_{S_j}(x)\right| \leq E = o(\sqrt{n})$, whp adversary can reconstruct $1 - o(1)$ fraction of the bits x_i.
- Proof idea: $A(S_1, a_1, \ldots, S_m, a_n)$ = any $\hat{x} \in \{0,1\}^n$ s.t. $\forall j \; \left|a_j - q_{S_j}(\hat{x})\right| \leq E$.

  (Show that whp, for all $\hat{x}$ that differs from x in a constant fraction of bits, $\exists j$ such that $\left|q_{S_j}(\hat{x}) - q_{S_j}(x)\right| > 2E$.)

Integer Programming Implementation

$A(S_1, a_1, \dots, S_m, a_n)$:

1. Find a vector $\hat{x} \in \mathbb{Z}^n$ such that:
   - $0 \leq \hat{x}_i \leq 1$ for all $i = 1, \dots, n$
   - $-E \leq a_j - \sum_{i \in S_j} \hat{x}_i \leq E$ for all $j = 1, \dots, m$
2. Output $\hat{x}$.

Problem: Can be computationally expensive ("NP-hard", exponential time in worst case)

Faster: Linear Programming Implementation

$A(S_1, a_1, \ldots, S_m, a_n)$:

1. Find a vector $\hat{x} \in \mathbb{R}^n$ such that:
   - $0 \leq \hat{x}_i \leq 1$ for all $i = 1, \ldots, n$
   - $-E \leq a_j - \sum_{i \in S_j} \hat{x}_i \leq E$ for all $j = 1, \ldots, m$
2. Output $\hat{x}$

Linear Programming Implementation for Average Error

$A(S_1, a_1, \dots, S_m, a_n)$:

1. Find vectors $\hat{x} \in \mathbb{R}^n$ and $E \in \mathbb{R}^m$
   – Minimizing $\sum_{j=1}^{m} E_j$ and such that
   – $0 \leq \hat{x}_i \leq 1$ for all $i = 1, \dots, n$
   – $-E_j \leq a_j - \sum_{i \in S_j} \hat{x}_i \leq E_j$ for all $j = 1, \dots, m$
2. Output $\mathrm{round}(\hat{x})$.

Least-Squares Implementation for MSE

$A(S_1, a_1, \dots, S_m, a_n)$:

1. Find vector $\hat{x} \in \mathbb{R}^n$ minimizing

$$\sum_{j=1}^{m} \left(a_j - \sum_{i \in S_j} \hat{x}_i \right)^2 = \|a - M_S \hat{x}\|^2$$

2. Output $\mathrm{round}(\hat{x})$.

Also works for random S_j's, and is much faster than LP!

On the Level of Accuracy

- The theorems require the error per statistic to be $o(\sqrt{n})$. This is necessary for reconstructing almost all of x.

- Q: What is significant about the threshold of $\sqrt{n}$?
  - If dataset is a random sample of size n from a larger population, the standard deviation of a count query is $O(\sqrt{n})$.
  - Reconstruction attacks ⇒ if we want to release many ($> n$) arbitrary or random counts, then we need introduce error at least as large as the sampling error to protect privacy.

How to Make Subset Sum Queries?

- Stylized set-up:
  - Dataset $x \in \{0,1\}^n$.
  - (Known) person i has sensitive bit x_i.
  - Adversary gets $a_S \approx q_S(x) = \sum_{i \in S} x_i$ for various $S \subseteq [n]$.
- Q: How to attack if the subjects aren't numbered w/ ID's?
  - If we know the set of people but not their IDs? (e.g. current Harvard students)
  - If we only know the size n of the dataset?

ID	US?
1	1
2	0
3	0
4	1
$\vdots$	$\vdots$
n	1

Overall Message

- Every statistic released yields a (hard or soft) constraint on the dataset.
  - Sometimes have nonlinear or logical constraints $\Rightarrow$ use fancier solvers (e.g. SAT or SMT solvers)
- Releasing too many statistics with too much accuracy necessarily determines almost the entire dataset.
- This works in theory and in practice (see readings, ps2).